

# C.U.SHAH UNIVERSITY

## Summer Examination-2017

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma(All)

Semester : 2

Date : 04/05/2017

Time : 02:00 To 05:00

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions:**

**(14)**

- a)  $|i + 3j - 2k| = \underline{\hspace{2cm}}$ .
- (a)  $\sqrt{14}$  (b) 14 (c) 2 (d) None of these
- b) If  $(1, -2, 3) \cdot (4, 5, k) = 0$  then  $k = \underline{\hspace{2cm}}$ .
- (a) -2 (b) 2 (c) 1/2 (d) None of these
- c) If  $\theta$  is the angle between the vectors  $\vec{x}$  and  $\vec{y}$  then  $\sin\theta = \underline{\hspace{2cm}}$
- (a)  $\frac{\vec{x} \times \vec{y}}{|\vec{x} \times \vec{y}|}$  (b)  $\frac{|\vec{x} \times \vec{y}|}{|\vec{x}||\vec{y}|}$  (c)  $\frac{\vec{x} \cdot \vec{y}}{|\vec{x} \times \vec{y}|}$  (d) None of these
- d) If  $x = (1, 1, 1)$  and  $y = (1, 0, 0)$  then  $x - y = \underline{\hspace{2cm}}$ .
- (a) (0,1,0) (b) (0,0,1) (c) (1,0,0) (d) None of these
- e)  $\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \underline{\hspace{2cm}}$
- (a) m/n (b) n/m (c) 1 (d) None of these
- f)  $\lim_{x \rightarrow 0} \left(1 + \frac{2}{x}\right)^x = \underline{\hspace{2cm}}$
- (a)  $e^2$  (b) e (c)  $e^{1/2}$  (d) None of these
- g)  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = \underline{\hspace{2cm}}$
- (a)  $ax^{a-1}$  (b)  $nx^{n-1}$  (c)  $na^{n-1}$  (d) None of these
- h)  $\frac{d(x^2 + 2x + 7)}{dx} = \underline{\hspace{2cm}}$ .
- (a)  $x+2$  (b)  $2x+2$  (c) 2 (d) None of these
- i)  $\frac{d(\sec^2 x - \tan^2 x)}{dx} = \underline{\hspace{2cm}}$ .
- (a) 2 (b) 1 (c) 0 (d) None of these



- j)  $\frac{d(6^x)}{dx} = \underline{\hspace{2cm}}$   
 (a)  $6^x \log_e 6$  (b)  $\log_e 6$  (c)  $x^6 \log_e 6$  (d)  $6^x \log_e x$
- k) If  $f(x) = \log \sqrt{x^2 + 1}$  then  $f'(0) = \underline{\hspace{2cm}}$   
 (a)  $\log 2$  (b)  $\frac{1}{2} \log 2$  (c)  $2 \log 2$  (d)  $0$
- l)  $\int \frac{-1}{x^2 + 1} dx = \underline{\hspace{2cm}}$   
 (a)  $\tan^{-1} x + c$  (b)  $\sin^{-1} x + c$  (c)  $\cos^{-1} x + c$  (d)  $\cot^{-1} x + c$
- m)  $\int \frac{1}{\sqrt{x^2 + 4}} dx = \underline{\hspace{2cm}}$   
 (a)  $\cot^{-1} \frac{x}{2} + c$  (b)  $\tan^{-1} \frac{x}{2} + c$  (c)  $\log \left| x + \sqrt{x^2 + 4} \right| + c$  (d) none of these
- n)  $\int_4^5 \frac{1}{x} dx = \underline{\hspace{2cm}}$   
 (a)  $\log \frac{4}{5}$  (b)  $\log \frac{5}{4}$  (c)  $\log 10$  (d) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

- a) If  $\bar{a} = i + 2j - k$ ,  $\bar{b} = 3i + j + 2k$  and  $\bar{c} = -2i - j + 5k$  then find  $|2\bar{a} + 3\bar{b} - \bar{c}|$ . (5)
- b) If  $\bar{a} = i + j + k$  and  $\bar{b} = 2i - 2j + k$  then find unit vector perpendicular to  $\bar{a}$  and  $\bar{b}$ . (5)
- c) Evaluate:  $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$  (4)

**Q-3 Attempt all questions (14)**

- a) Prove that  $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x} = \frac{1}{\sqrt{a}}$ . (5)
- b) Evaluate:  $\lim_{x \rightarrow 0} \frac{2(5^x) + 3(2^x) - 5}{x}$  (5)
- c) Find  $\frac{dy}{dx}$  if  $y = \frac{a + b \sin x}{a \sin x + b}$  (4)

**Q-4 Attempt all questions (14)**

- a) The equation of motion of a particle is  $S = 2t^3 + 3t^2 - 12t + 5$ . Find velocity at  $t = 0$  and acceleration at  $t = 1$ . (5)
- b) Find  $\frac{dy}{dx}$  if  $y = \log \left( \frac{\sin x}{1 + \cos x} \right)$ . (5)
- c) For what value of  $m$ , vectors  $m\bar{j} + 2\bar{i} + \bar{k}$  and  $2\bar{i} + 4\bar{j} + 5\bar{k}$  are perpendicular (4)



to each other?

**Q-5 Attempt all questions (14)**

a) Prove that angle between two vectors  $i + 2j$  and  $i + j + 3k$  is  $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$ . (5)

b) Find  $\frac{dy}{dx}$  if  $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$  and  $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$ . (5)

c) Evaluate:  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$  (4)

**Q-6 Attempt all questions (14)**

a) Forces  $\vec{F}_1 = i + 2j - 3k$  and  $\vec{F}_2 = i - j + 2k$  act on a particle under the influence of these forces, particle moves from point  $(3, 1, 2)$  to  $(1, 3, -1)$ . Find the work done. (5)

b) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$  (5)

c) Find derivative of  $f(x) = e^x$  using definition. (4)

**Q-7 Attempt all questions (14)**

a) Evaluate:  $\int x \sin x dx$  (5)

b) Find area of region bounded between  $y = x^2$ , X-axis,  $x = 1$  and  $x = 2$ . (5)

c) If  $\vec{a} = (2, -3, -1)$  and  $\vec{b} = (1, 4, -3)$  then find  $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ . (4)

**Q-8 Attempt all questions (14)**

a) If  $f'(x) = 4x^2 + 6x - 3$  and  $f(1) = 2$  then find function  $f(x)$ . (5)

b) Find  $\frac{dy}{dx}$  if  $y = (\sin x)^{\tan x}$ . (5)

c) Evaluate:  $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$  (4)

